

# Measurement of the Muller matrix for painted surfaces with a kind of scatterometer

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## ABSTRACT

The polarized light scattered by the surface of a material contains information that can be used to describe the properties of the surfaces. Polarized Bidirectional Reflectance Distribution Function (BRDF) is one of the most important factors used to represent the property of the surface. It uses a 4×4 matrix (Mueller matrix) to describe the properties of the light scattered from the surface. In order to measure the Mueller matrix of the samples, a new three axis automated scatterometer has been developed to measure the Mueller matrix of painted surfaces. It can do measurement at any illumination and viewing geometric of the hemisphere and it is more convenient for far-field measurement is presented. The design of the instrument is different to the traditional scatterometer. The significant characteristic of the instrument is that the detector and polarization analyzer are fixed, while the source and the incident optical elements rotate on a stage together. All the possible incident and viewing positions can be reached through the rotation of three motors. The rotations of the motors are fed back through photoelectric- encoders, the “closed loop” control mode ensured the precision of the position. Through coordinate transformations, the measurement in three dimensions can be simplified in two dimensional form, the details of the coordinate transformations will be described in detail in this paper. The dual-rotating retarders method is used to modulate polarizing and analyzing optics. Two retarders rotate synchronously at angular speed and respectively. For every position, 16 measurements were done, and the Discrete Fourier Transform (DFT) method is used to retrieve the Mueller matrix of the sample. Discrete Fourier Transform (DFT) method is used to retrieve the Mueller matrix of the sample. The results of out-plane polarized bidirectional reflectance distribution function for samples coated with different paints are presented.

**Keywords:** Polarization; Scatterometer; Muller matrix; Bidirectional Reflectance Distribution Function (BRDF);

## 1. INTRODUCTION

The conventional method that used to describe the reflected component of the radiance from a surface is the Bidirectional Reflectance Distribution Function (BRDF) defined by Nicodemus<sup>[1]</sup> and the BRDF is defined as:

$$f_r(\theta_i, \phi_i, \theta_r, \phi_r) = \frac{dL_r(\theta_i, \phi_i, \theta_r, \phi_r)}{dH_i(\theta_i, \phi_i)} (sr^{-1}) \quad (1)$$

Measurement of BRDF<sup>[2-5]</sup> and the analysis in terms of BRDF are commonplace. Unfortunately, the BRDF description does not properly account for the polarization characteristics of the reflected radiation. A key factor that is used to describe painted objects is the polarized BRDF which contains more information than the unpolarized BRDF and can be used for target surveillance and recognition system<sup>[6,7]</sup>. The represents of the PBRDF using a 4×4 matrix (Mueller matrix) to describe the properties of the light scattered from the surface.

NIST<sup>[8-10]</sup> and US naval research Lab<sup>[11]</sup> build measurement system respectively to get the Mueller matrix of the surfaces. It is common to measure the Mueller matrix within the plane of incidence (simplified as in-plane in the following), for it requires the least optical hardware and it is easy to build the system. The measurement for Mueller matrix out of incident plane (simplified as out-plane in the following) usually more complicated than the in-plane.

In this paper, we describe a new instrument that can measure the in-plane and out-plane Mueller matrix for the painted surface. The design of the instrument is different to the traditional scatterometer, the significant characteristic of the

instrument is that the detector and polarization analyzer are fixed, while the source and the incident optical elements rotate on a stage together. All the possible incident and viewing positions can be reached through the rotation of three motors. The rotations of the motors are fed back through photoelectric- encoders, the “closed loop” control mode ensured the precision of the position. Through coordinate transformations, the measurement in three dimensions can be simplified in two dimensional form, the details of the coordinate transformations will be described in detail in this paper. Two retarders rotate synchronously at angular speed  $\omega$  and  $4\omega$  respectively. For every position, 16 measurements were done, and the Discrete Fourier Transform (DFT) method is used to retrieve the Mueller matrix of the sample. Finally, the Mueller matrix results of in-plane and out-plane for several samples with different paint are given in this paper.

## 2. MUELLER MATRIX MEASUREMENT SYSTEM

Figure 1 is the experimental arrangement for Mueller matrix measurement. Light from a laser (current wavelength  $1.06 \mu\text{m}$ ), passes through a beam expander system with a chopper in the focal plane, polarizer, an interchangeable retarder, two reflection prisms, before being focused on the center of the sample. The rotations of waveplate 1 and waveplate 2 are controlled by motor D and motor E respectively. Polarizer 1 and polarizer 2 are mounted on a manual rotation stage, during the measurement two polarizers are fixed. The three-axis (Motor A, Motor B and Motor C) system can do measurement for any different combinations of incidence and viewing angles. Three photoelectric encoders are used to record the position of three motors. Before the measurement, the calibration of the position of the motors is performed by the encoders. The polarization analysis system is comprised of polarizer 2 and waveplate 2 in the receiving part. Following the polarization system is the PMT (photomultiplier tube). During the measurement, the center of the sample is illuminated, it should be ensured that the viewing field of the detector bigger than the illuminated area at any viewing angle. For the limitation of structure of the system, in the retro-direction there is about  $\pm 1^\circ$  occultation. The character of the previous NIST design is that the receiving part fixed on the moving stage and the emitting part is fixed [8]. In our configuration, we exchange the position of the emitting part and the receiving part, the receiving part is mounted on an optical platform (is not move in the measurement), which is different with NIST design (for details of the placement see figure 1). In this kind of configuration, the distance between the receiving part and the sample can be “flexibly changed”, other optical elements can be added in the optical path easily, so it is much easier to realize both far-field and near-field measurements. The far-field result is important for “reduced scale model” measurements. It is hard to realize this kind of measurement with the NIST configuration (there is space limitation in the rotation arm). The NIST design is much more suitable for measurement with different incident angle and constant receiving angle. In practical applications, the reflection parameters varying with  $\theta_r, \phi_r$  (in the receiving part) is more concerned, the receiving angle is constant mode is not suitable. In our configuration, when the incident angle is constant and receiving angle continued changing mode is needed, it is much easier to realize both mathematically and for data processing.

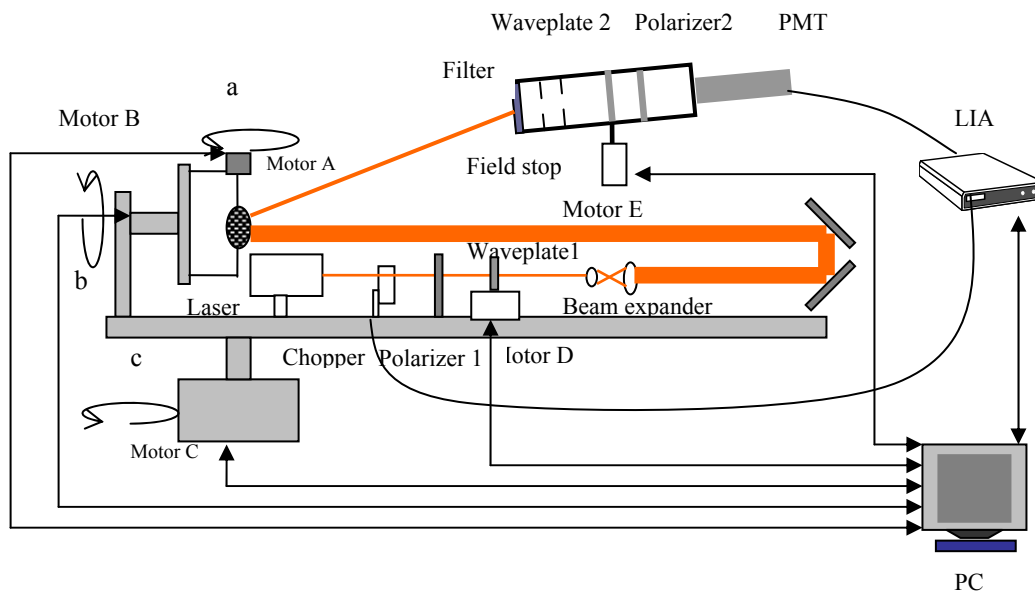


Figure1 Schematic diagram of the PBRDF Scatterometer

### 3. MOTOR CONTROL COORDINATE TRANSFORMATION

The coordinate system established on the surface of the sample is shown in figure 2.  $\theta, \phi$  represent the zenith and azimuth angle respectively,  $Z$  represents the surface normal. The subscript  $r$  and  $i$  represent the reflection light and the incident light respectively.  $\vec{k}_i$  is the unit vector in the specific light source.  $\vec{k}_r$  is the unit vector in the direction of the viewer. They are related by:

$$\vec{p}_i = \vec{k}_i \times \vec{s}_i, \quad \vec{p}_r = \vec{k}_r \times \vec{s}_r \quad (2)$$

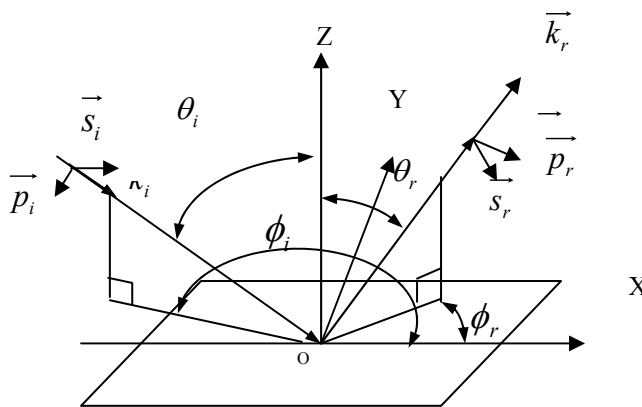


Figure 2 PBRDF coordination for Mueller matrix measurement

In order to realize the 3 dimensions space measurement in the 2 dimensional space, it is necessary to convert three motor (Motor A, Motor B and Motor C) rotation angles a, b, c to the sample coordinate system. The control mode of the three motor is the following:

$$\begin{cases} \theta_i = a \\ \phi_i = 90^\circ - \phi_0 \\ \cos \theta_r = \cos a \cdot \cos c - \sin a \cdot \cos b \cdot \sin c \\ \cos \phi'_0 = \sin b \cdot \sin c / \sin \theta_r \\ \phi_r = \phi'_0 - \phi_0 \end{cases} \quad (3)$$

Where a, b, c are the rotation angles of Motor A, Motor B and Motor C respectively,  $\theta_i$  is the incident zenith angle,  $\theta_r$  is the viewing zenith angle,  $\phi_r$  is the viewing azimuth angle,  $\phi_0$  is the angle between the axis of motor A and the pre-defined position of the sample, which should not change if the sample is mounted on the holder, and if the sample is isotropic it is no required to consider the  $\phi_0$ .  $\phi'_0$  is an auxiliary angle used in the coordinate transformation.

In the polarization measurement, we work under the assumption that the most natural condition is that the  $\vec{s}$  ( $\vec{p}$ ) vectors are perpendicular (parallel) to the incidence plane or the outgoing plane. When measuring the polarization properties of out-plane optical scattered light, the polarization coordinate transformation is needed:

$$\begin{cases} \psi_i = b \\ \cos \psi_r = \frac{\cos a - \cos \theta_r \cos c}{\sin \theta_r \sin c} \end{cases} \quad (4)$$

Where  $\psi_i$  is the angle that the polarization coordinates need to rotate in the incident part,  $\psi_r$  is the angle that the polarization coordinates need to rotate in the polarization analysis part.

#### 4. MUELLER MATRIX MEASUREMENT FOR PAINTED SURFACE

In polarization analysis of the samples, the properties of the scattered light can be described by a Stokes vector, as the follows:

$$\Phi_{stokesr}(\theta_i, \phi_i; \theta_r, \phi_r) = \begin{pmatrix} \Phi_{rH}(\theta_i, \phi_i; \theta_r, \phi_r) + \Phi_{rV}(\theta_i, \phi_i; \theta_r, \phi_r) \\ \Phi_{rH}(\theta_i, \phi_i; \theta_r, \phi_r) - \Phi_{rV}(\theta_i, \phi_i; \theta_r, \phi_r) \\ \Phi_{r45}(\theta_i, \phi_i; \theta_r, \phi_r) - \Phi_{r135}(\theta_i, \phi_i; \theta_r, \phi_r) \\ \Phi_{rR}(\theta_i, \phi_i; \theta_r, \phi_r) - \Phi_{rL}(\theta_i, \phi_i; \theta_r, \phi_r) \end{pmatrix} \quad (5)$$

$\Phi_{stokesr}$  is the Stokes vector of the scattered light,  $\Phi_{stokesi}$  is the Stokes vector of the incident light,  $M(\theta_i, \phi_i; \theta_r, \phi_r)$  is the matrix used to character the polarized properties of the optical system, related as the follows:

$$\Phi_{stokesr}(\theta_i, \phi_i; \theta_r, \phi_r) = M(\theta_i, \phi_i; \theta_r, \phi_r) \Phi_{stokesi} \quad (6)$$

The dual-rotating retarders method<sup>[12]</sup> is used to calculate the Mueller matrix. Two retarders rotating synchronously at angular speed  $\omega$  and  $4\omega$ . The output signal is in proportion to the intensity, which is the first element of the Stokes vector, expressed in the following formulation:

$$\Phi_{stokesr}^1(\omega, 4\omega) = A_1 M(\theta_i, \phi_i; \theta_r, \phi_r) P$$

$$\begin{aligned}
&= \frac{1}{2} [1, \cos 16\omega, \sin 16\omega, 0] \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \\ m_{30} & m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} 1 \\ -\cos 4\omega \\ -\sin 4\omega \\ 0 \end{bmatrix} \\
&= \\
&\frac{1}{2} [m_{00} + m_{01} \cos 4\omega + m_{02} \sin 4\omega - m_{10} \cos 16\omega - \frac{1}{2} m_{11} (\cos 12\omega + \cos 20\omega) - \\
&\frac{1}{2} m_{12} (\sin 20\omega - \sin 12\omega) - m_{20} \sin 16\omega - \frac{1}{2} m_{21} (\sin 20\omega + \sin 12\omega) - \frac{1}{2} m_{22} (\cos 12\omega - \cos 20\omega)] \quad (7)
\end{aligned}$$

Where  $A_1$  is the first row of the Mueller matrix for waveplate 2, P is the Stokes vector of waveplate 1, M is the Mueller matrix of the sample. In expression (7) there is no element of last row and last column for M, so the Mueller matrix can be simplified to a  $3 \times 3$  form. The advantage of this method is that the system measurement error can be reduced [9].

$\Phi_{stokesr}^1(\omega, 4\omega)$  can be written in terms of its Fourier spectrum:

$$\Phi_{stokesr}^1(\omega, 4\omega) = K \sum_j (c_j \cos(j\omega) + s_j \sin(j\omega)) \quad (8)$$

$c_j$  and  $s_j$  can be measured by Discrete Fourier Transform (DFT) method. K is the coefficient of the detector. The Mueller matrix of the sample surface can be expressed using  $c_j$  and  $s_j$  as the following:

$$M(\theta_i, \phi_i; \theta_r, \phi_r) = \begin{pmatrix} c_0 & c_4 & -s_4 & * \\ -c_{16} & -c_{12} - c_{20} & s_{20} - s_{12} & * \\ -s_{16} & -s_{12} - s_{20} & c_{20} - c_{12} & * \\ * & * & * & * \end{pmatrix} \quad (9)$$

Measurement time varies with different sampling position. Generally, the sampling intervals are  $10^\circ$ , the time cost for the whole hemisphere measurement is about 4 hours.

## 5. EXPERIMENTAL RESULTS AND ANALYSIS

In the following part we will introduce some experimental results for several samples with different color painted on steel panel using this scatterometer. The wavelength of the laser is  $1.06 \mu\text{m}$ , Figure 3, Figure 4, Figure 5, Figure 6 are the Mueller matrix measurement results of out-plane for 5 different samples. In Figure 3, Figure 4, Figure 5, Figure 6 is the measurement result of  $M_{ij}$  change with viewing azimuth angle when both the incidence zenith  $\theta_i$  and receiving zenith  $\theta_r$  are  $20^\circ$ . In Figure 7 the measurement results of  $M_{ij}$  change with viewing azimuth angle when both the incidence zenith  $\theta_i$  and receiving zenith  $\theta_r$  are  $30^\circ$ . The 5 samples have different color and roughness, the complex refractive index of refraction is more complicated. Although the interpretation of the data for scattering mechanics is difficult, the Muller matrix characteristics of out-plane for different painted surfaces are clearly seen. These characteristics of the painted surfaces can agree well with the model [13-16] and can be used as a reference for materials diagnosis and feature extraction.

In the application, it is found that the roughness plays an important role in the polarization measurement. The degree of polarization is one of the parameter used to character the depolarization abilities which can be calculated using the Muller matrix elements as the following:

$$P_L = \frac{f_{\max} - f_{\min}}{f_{\max} + f_{\min}} = \frac{\sqrt{(S_1^2 + S_2^2)}}{S_0} = \frac{\sqrt{(M_{10} - M_{11})^2 + (M_{20} - M_{21})^2}}{M_{00} - M_{01}} \quad (10)$$

Figure 8 is the out-plane results degree of polarization for 3 samples which have the same painting material but with different roughness. In this figure, the correlation lengths of three samples are 30-45  $\mu$ , 425-450  $\mu$ , 450-480  $\mu$  respectively. From the comparison we can found, the higher the roughness, the smaller the degree of polarization.

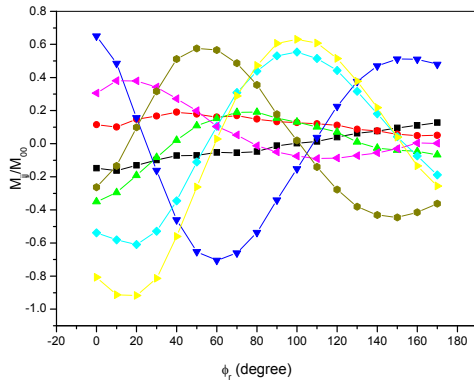


Figure 3 Mueller matrix elements  $M_{ij}$  normalized to  $M_{00}$  for sample1

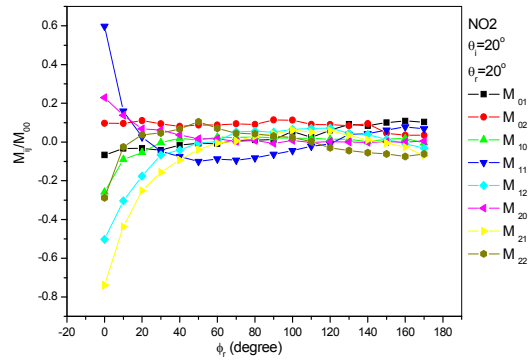


Figure 4 Mueller matrix elements  $M_{ij}$  normalized to  $M_{00}$  for sample2

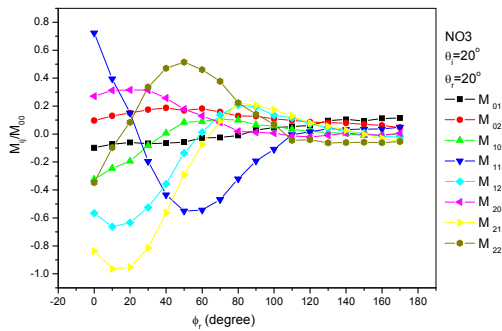


Figure 5 Mueller matrix elements  $M_{ij}$  normalized to  $M_{00}$  for sample3

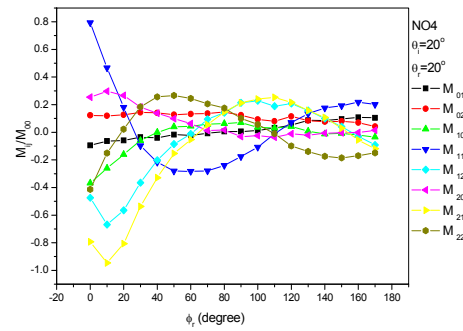


Figure 6 Mueller matrix elements  $M_{ij}$  normalized to  $M_{00}$  for sample4

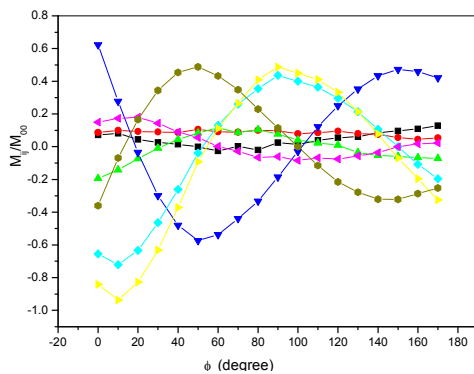


Figure 7 Mueller matrix elements  $M_{ij}$  normalized to  $M_{00}$  for sample5

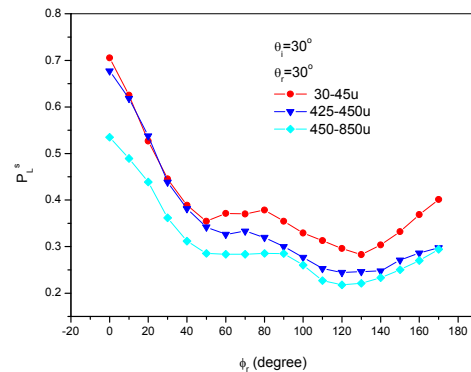


Figure 8 The degree of polarization for 3 samples with different roughness parameters change with azimuth angle

## 6. CONCLUSION

A new scatterometer used to measure the Mueller matrix of painted surface is reviewed in this previous paper which is very convenient for both the far-field and near field measurement. The scatterometer can get the Mueller matrix for almost all the positions in the hemisphere. The controlling of the motors is described, and the algorithm used to get the Mueller matrix is given. The measurements results of 5 painted samples for out-plane are presented. For the samples painted with the same material but with different roughness, the degrees of polarization are different. If the polarized BRDF of other wavelengths are needed, the extra work we need to do is to change the Laser and the retarders, the control mode and the algorithm for Mueller matrix are the same.

## ACKNOWLEDGEMENTS

This work was supported by State Key Development Program for Basic Research of China (61341020201-1) and the Department of Science and Technology of Shandong Province (2008GG20005005).

## REFERENCES

1. F.E. Nicodemus, J.C. Richmond, and J.J. Hsia, Geometrical Considerations and Nomenclature for Reflectance, NATIONAL BUREAU OF STANDARDS, Ernest Ambler, (1977) .
2. B. T. McGuckin, D. A. Haner, R. T. Menzies, et al. "Directional reflectance characterization facility and measurement methodology", APPLIED OPTICS, Vol. 35, No. 24,4827–4834(1996).
3. Wei Qingnong, Liu Jianguo and Jiang Rongxi, "Measurement Method of Absolute Bidirectional Reflectance Distribution Function", Acta Optica Sinica, Vol.16, No.10, 1425–1430(1996).
4. Wu Zhensen, Han Xiange, Zhang Xiangdong, et al. "Experimental Study on Bidirectional Reflectance Distribution Function of Laser Scattering from Various Rough Surfaces", Acta Optica Sinica, Vol.16, No.3, 262–268(1996).
5. Zhang Baishun, Liu Wenqing, Wei Qingnong, et al. "Analysis of scattering characteristic of the sample based on BRDF experiment measurements", Optical Technique, Vol.32 No.2, 180–182(2006).
6. Cornell S. L. Chun, Firooz A.Sadjadi, "Target Recognition Study Using Polarimetric Laser Radar", Proceedings of SPIE 5426,274–284(2004).
7. Richard G. Priest, Thomas A. Germer, "Polarimetric BRDF in the Microfacet Model: Theory and Measurement", Published in Proceedings of the 2000 Meeting of the Military Sensing Symposia Specialty Group on Passive Sensors, Vol.1,169–181(2000).
8. T. A. Germer, and C. C. Asmail, "A goniometric optical scatter instrument for bidirectional reflectance distribution function measurements with out-of-plane and polarimetry capabilities", Proc. SPIE 3141, 220–231(1997)..
9. Thomas A. Germer and Clara C. Asmail, "Goniometric optical scatter instrument for out-of-plane ellipsometry measurements", Vol.70, No.9, 3688–3695(1999).
10. Thomas A. Germer, Clara C. Asmail, "Polarization of light scattered by microrough surfaces and subsurface defects". J. Opt. Soc. Am. Vol.16, No. 6, 1326–1332(1999).
11. Steven R. Meier and Richard G. Priest, "Mueller Matrix Measurements of Black and White Materials in the Infrared", Proceedings of SPIE 4133, 82–91(2000).
12. R. M. A. Azzam, "Photopolarimetric measurement of the Mueller matrix by Fourier analysis of a single detected signal", OPTICS LETTERS, Vol. 2, No. 6, 148–150(1978).
13. Feng Weiwei, Wei Qingnong, Wang Shimei, et al. "Numerical simulation of rough surface Bidirectional Reflectance Distribution Function (BRDF)", Proc. SPIE 6723, 672314 (2007)
14. Feng Weiwei, Wei Qingnong, Wang Shimei, et al. "Numerical simulation of polarized bidirectional reflectance distribution function (BRDF) based on micro-facet model", Proc. SPIE Vol. 6622, 66220A (2008)
15. Feng Weiwei, Wei Qingnong, Wang Shimei, et al, "Study of polarized Bidirectional Reflectance Distribution Function model for painted surfaces", Acta Optica Sinica. Vol.28,NO.2,290-294(2008).
16. Feng Weiwei, Wei Qingnong, Wang Shimei, et al, "Optimized modeling of polarized BRDF based on hybrid genetic algorithm for painted surfaces", Infrared and Laser Engineering, Vol.37,NO.4, 743–747 (2008).